Elimination of End Losses in an MHD Induction Generator

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Introduction

A PARTICULAR difficulty encountered in the design of MHD induction generators is the elimination of end losses. These losses have a marked effect on efficiency, and arise from currents induced by fringing fields associated with the finite length of the excitation current sheet.[1,2] Such currents may be physically eliminated by the insertion of nonconducting baffles in the end sections; however, it is then necessary to modify the electromagnetic fields in these regions so as to maintain an ideal waveform in the device. As first proposed by P. F. Meader†, this may be accomplished by the use of end coils.

Previous discussions of this problem have been somewhat obscure,[3,4] and it is the purpose of this analysis to provide a simple determination of the end coil currents necessary to accomplish the desired matching. Specifically, the generator design in question is shown on Fig. 1.

On this figure the mean radius, from the center line of the generator to the center line of the flow gap, is r_m. The distance from the center line of the flow gap to the current sheets is r_0, and the overall length of the generator is L.

The axial coordinate x is taken to be zero at the entrance section, and the local-flow gap area is proportional to $A(x)$ (to allow for compressibility).

The traveling wave excitation current sheet is characterized by a frequency ω and a wave number k, and has an amplitude I_0 current per unit length.

It shall be assumed that the relative dimensioning of the device is such that the ratios $\epsilon_r = r_0/r_m$, $\epsilon = kr_0$, and $\epsilon_x = 1/kL$ are all small while the excitation wavelength ($\propto 1/k$) is of the same order of magnitude as the mean radius (r_m).

In the limit, where ϵ_r is small (to zeroth-order in ϵ_r), this problem may be treated in terms of the Cartesian variables x_1 and y_1 where

$$x_1 = kx \tag{1}$$

and

$$r = r_m (1 + \epsilon_r y) \tag{2}$$

with

$$y_1 = \epsilon y \tag{3}$$

Dependence on azimuthal angle is eliminated by symmetry.

Electromagnetic Fields in the Generator

No attempt will be made here to analyze the actual flow or electromagnetic field configurations in such a generator. For the purposes of this discussion only the cross-channel magnetic field and the circumferential electric field are important. The approximate behavior of these fields is determined by Faraday's Law of Induction which in the present context may be written as

$$\nabla_{x_1}^{y_1} \times \mathbf{E}' = -\partial \mathbf{B}'/\partial \tau \tag{4}$$

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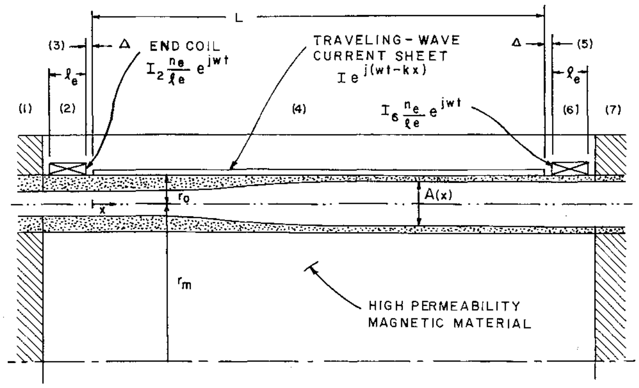

Fig. 1 Sectioned view of simplified MHD induction generator.

where $\mathbf{B} = B_0\mathbf{B}'$, $\mathbf{E} = (\omega/k)B_0\mathbf{E}'$, $\tau = \omega t$ and $(\nabla_{x_1}^{y_1} \times)$ is a vector differential operator in the Cartesian space (x_1,y_1,θ).

In order to insure that the major interaction in the generator be with the axial velocity, the magnetic field is taken as

$$\mathbf{B} = B_0[b_x, B_y'(x_{1,\tau}) + b_y] \tag{5}$$

and consistently the electric field takes the form

$$\mathbf{E}_\theta = (\omega/k)B_0[E_\theta'(x_{1,\tau}) + e_\theta] \tag{6}$$

where $|B_y'| \gg |\mathbf{b}|$ and $|E_\theta'| \gg |e_\theta|$.

With these definitions Eq. (4) implies

$$\partial E_\theta'/\partial x_1 = \partial B_y'/\partial \tau \tag{7}$$

and if

$$B_y = B_0 e^{j(\omega t - kx)} \tag{8}$$

then

$$E_\theta = -(\omega/k)B_0 e^{j(\omega t - kx)} \tag{9}$$

The perturbation fields $\mathbf{b}$ and e_θ may be calculated[5] as power series in ϵ, and for the purposes of this analysis may be ignored.

It should be noted that when a magnetic waveform is specified, the current in the excitation current sheet becomes an unknown and must be found by applying suitable boundary conditions after the complete flow in the generator has been determined. This is the inverse of the normal boundary value problem.

End Coil Currents

Referring again to Fig. 1, there are seven distinct regions identified in which the boundary conditions on the electromagnetic fields are different. The fields in region 4 have been mentioned previously. It is therefore necessary to solve for the fields in the remaining 6 regions and match both the magnetic and electric fields at the region boundaries.

Because of the assumed presence of nonconducting baffles in the end regions, no currents may flow there and the electromagnetic fields are uncoupled from the flowfield. Consequently, the electromagnetic problem reduces to the solution of a potential problem given by

$$(\nabla_{x_1}^{y_1})^2 \Psi = 0 \tag{10}$$

where

$$B_x = \partial\Psi/\partial y_1,\ B_y = -\partial\Psi/\partial x_1 \tag{11}$$

and

$$E_\theta = -(1/k)\partial\Psi/\partial t \tag{12}$$

The appropriate boundary conditions on Ψ in the various regions are a) regions 2 and 6

$$\left.\frac{\partial\Psi_{2,6}}{\partial y_1}\right|_{y_1=\epsilon} = \mu\frac{N_e}{l_e}\begin{bmatrix}I_2\\I_6\end{bmatrix}e^{j\omega t}$$
$$\left.\frac{\partial\Psi_{2,6}}{\partial y_1}\right|_{y_1=-\epsilon} = 0 \tag{13}$$

where N_e is the number of turns of wire in an end coil of length l_e carrying a current $I_{2,6}$ (amps), and b) regions 1, 3, 5, and 7

$$\left.\frac{\partial\Psi_{1,3,5,7}}{\partial y_1}\right|_{y_1=\pm\epsilon} = 0 \tag{14}$$

Solving Eq. (10) subject to the aforementioned boundary conditions gives a) in regions 2 and 6

$$\Psi_{2,6} = \frac{\mu N_e}{2\epsilon l_e}\begin{bmatrix}I_2\\I_6\end{bmatrix}e^{j\omega t}\left[\epsilon^2\left(\frac{y^2}{2}+y\right) - \left(\frac{x_1{}^2}{2} + a_1{}^{(\pm)}x_1 + a_2{}^{(\pm)}\right)\right] \tag{15}$$

b) in regions 3 and 5,

$$\Psi_{3,5} = -\,(b_1{}^{(\pm)}\,x_1 + b_2{}^{(\pm)})\,e^{j\omega t} \tag{16}$$

and c) in regions 1 and 7

$$\Psi_{1,7} = -\,c_1{}^{(\pm)}e^{j\omega t} \tag{17}$$

These solutions may be varified by a direct application of the integral form of Ampere's Law assuming, as is consistent with Eqs. (8) and (9), that B_y is independent of y. The general solution in regions 1 and 7 is identical to that in regions 3–5; however, since the former are semi-infinite, and all fields must be bounded, Ψ cannot be linear in x_1 there and hence, reduces to a simple constant.

Also, consistent with the accuracy of the solutions in region 4 (zeroth-order in ϵ) the y dependence of Ψ in regions 2 and 6 (second-order in ϵ) may be neglected. Then the cross-channel magnetic field and the azimuthal electric field may be determined from Eqs. (11) and (12).

Matching these fields in x_1 at the several region boundaries leads to twelve algebraic equations in the twelve unknowns $a_1{}^{(\pm)}$, $a_2{}^{(\pm)}$, $b_1{}^{(\pm)}$, $c_1{}^{(\pm)}$ and I_2 and I_6. Choosing I_2 and I_6 arbitrarily leads to an overspecified system of equations. The two missing constants, needed to close this system of equations, describe additional fields inside the generator and correspond to the major components of the undesirable end solutions.[1,2] It is only when I_2 and I_6 take on precisely the values determined by the previously mentioned matching procedure that these additional fields are mathematically unnecessary and are, in fact, absent in the generator.

The magnitudes of the desired end currents determined by this procedure are

$$I_2 = 2\epsilon B_0/k\mu N_e \tag{18}$$

and

$$I_6 = -\,2\epsilon B_0 e^{-jkL}/k\mu N_e \tag{19}$$

If the generator is so designed as to have an integral number of wavelengths λ of the magnetic field in the length L then

$$k = 2\pi/\lambda \text{ and } e^{-jkL} = 1 \tag{20}$$

For this case

$$\begin{bmatrix}I_2\\I_6\end{bmatrix} = \pm\frac{2\epsilon B_0}{k\mu N_e} = \pm\frac{2r_0B_0}{\mu N_e} \tag{21}$$

References

[1] Fanucci, J. B. et al., "Fringing Effects in an A.C. MHD Generator," *Engineering Aspects of Magnetohydrodynamics*, edited by Mather and Sutton, Gordon Breach, New York, 1964, pp. 329–343.

[2] Sudan, R. N., "Interaction of a Conducting Fluid Stream With a Traveling Wave of Magnetic Field of Finite Extension," *Journal of Applied Physics*, Vol. 31, No. 3, 1963, pp. 641–650.

[3] Moody, J. R., "A Self-Excited MHD Induction Generator," M.A. thesis, June 1965, Brown University, Providence, R.I.

[4] Jester, J. L. and Fanucci, J. B., "Characteristics of a Finite-Length MHD Traveling Wave Cylindrical Accelerator or Generator," *AIAA Journal*, Vol. 7, No. 5, May 1969, pp. 864–870.

[5] Lessmann, R. C., "An Analysis and Design Study of the Compressible, Traveling-Wave, Magnetohydrodynamic, Induction Generator," Ph.D. dissertation, June 1969, Brown University, Providence, R.I.

Analysis of Short Beams

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Nomenclature

A	= area of cross section of the beam
b,d	= breadth and depth of the rectangular beam
b_2,B_{22},C_{22}	= cross-sectional constants
D	= slenderness ratio = L/d
D^*	= L/nd
E	= Young's modulus
G	= shear modulus
I	= moment of inertia
k	= $(E/G)^{1/2}$
L	= length of the beam
n	= number of half waves into which the column buckles
P	= axial load
P_E	= Euler load
q	= transverse loading on the beam
U	= strain energy
v,w	= displacements along x and y directions, respectively
v_b	= transverse deflection caused by bending
v_s	= $v - v_b$
V	= nondimensional tip deflection = $3vEI/P_tL^3$
W	= work
w_n	= nth warp function
y,z	= Car esian coordinates
β	= shear coefficient
σ_z,σ_{zy}	= direct and shear stresses
ϕ_n	= axial variation of nth warp function
λ	= P/P_E
$'$	= denotes differentiation with respect to z

Introduction

THE beam may be defined as a slender structural component with one of its dimensions much larger than the other two. Its structural behaviour may be satisfactorily approximated by the elementary theory of bending as long as the slenderness ratio is sufficiently large. But for short beams, Euler's theory is inadequate because of the substantial influence of the secondary effects. In 1921, Timoshenko[1] extended the domain of validity of the beam theory for vibrations by incorporating the effect of transverse shear into the differential equation. Later this theory was extended for the stability[2] and static[3] analysis of beams. Recently the author has proposed a new formulation for the vibration analysis of short beams.[4] In this Note, we extend this formulation for the static and stability analysis of short beams and study some typical examples. A feature noted in Ref. 4, namely the use of refined shear coefficient in Timoshenko theory leading to increased discrepancies, is also noticed in the static and stability analysis of beams.

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